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Combined Stark-Zeeman effect for relativistic hydrogen-like ions moving in a transverse electric field and under planar channeling conditions

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Summary. — The motion of the ultra-relativistic hydrogen-like ion is considered in the transverse (*orthogonal to ion velocity*) electric field. In the rest frame of this ion both electric and magnetic fields exist. The combined influence of these fields on the electronic energy levels of the ion is investigated. It is shown that at high kinetic energy of the ion the electronic energy levels are shifted and are split not only due to the electric field but also due to the magnetic one. This phenomenon is named "the combined Stark-Zeeman effect". The influence of this effect on the electronic energy levels of planar channeled ions is also discussed.

PACS `61.85.+p` – Channeling phenomena (blocking, energy loss, etc.).
PACS `32.60.+i` – Zeeman and Stark effects.

1. – Introduction

It is well known the electronic energy levels of an ion placed in an electric field are shifted and are split due to the Stark effect. But when the ion is moving through a transverse electric field with relativistic velocity, in the rest frame of the ion both electric and magnetic fields exist. Hence, the position of electronic energy levels is changed not only by Stark effect but also due to Zeeman effect. The influence of magnetic field is negligible for non-relativistic and moderate ion energies, but it increases if the ion energy becomes relativistic. Let us define the situation, when the influence of the magnetic field is not negligible in comparison to the influence of the electric field, as "combined Stark-Zeeman effect" [1].

As expected, a new accelerator complex FAIR (Darmstadt, Germany) will provide the primary ion beam energy up to 30 GeV/u and this complex will open new perspectives for the studies in atomic physics (see, for example, in [2]). As we showed in [1] for the

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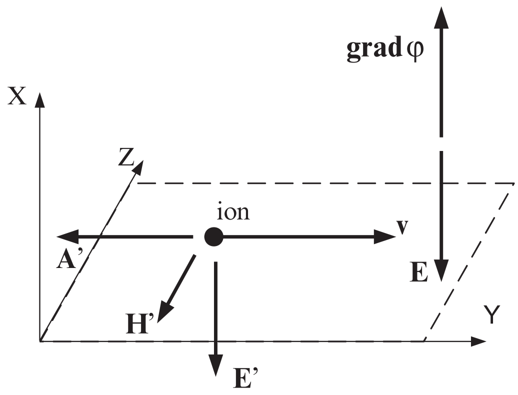

Fig. 1. – The laboratory frame XYZ and fields existing both in the laboratory frame and in the ion rest frame.

situation of planar channeling of relativistic hydrogen-like ions (H-like ions), the magnetic field existing in the ion rest frame at these energies influences essentially the electronic energy levels, and the combined Stark-Zeeman effect can change the condition to observe the resonant coherent ion excitation in a crystal.

In this work we consider the motion of ultra-relativistic H-like ions in a homogeneous electric field orthogonal to the ion velocity. The combined Stark-Zeeman effect for the planar channeled ions is also discussed in brief.

2. – The fields in the ion rest frame

Let us consider, first, the motion of the H-like ions in a homogeneous electric field directed as above pointed out. This field is described by the scalar potential ϕ and the strength of the electric field $\mathbf{E} = -\mathbf{grad}\phi$. In the ion rest frame the electric potential is $\phi' = \gamma\phi$, the strength of electric field is $\mathbf{E}' = -\mathbf{grad}\phi'$. Also in that frame the magnetic field is characterized both by the vector potential $\mathbf{A}' = -\gamma\mathbf{v}\phi/c$ and the strength $\mathbf{H}' = \mathrm{rot}\mathbf{A}'$. In these expressions γ is the relativistic factor and v is the ion velocity, c is the speed of light. Expressions above have shown that the vector $\mathbf{E}'$ is orthogonal to the velocity $\mathbf{v}$, the potential $\mathbf{A}'$ is directed opposite to the velocity $\mathbf{v}$ and the strength of the magnetic field $\mathbf{H}'$ is orthogonal both to the velocity $\mathbf{v}$ and to the strength of the electric field $\mathbf{E}'$. Let the electric field $\mathbf{E}$ be directed along the X-axis of the laboratory frame (see in fig. 1). Let us assume that the ion trajectory is rectilinear along the Y-axis. Hence, the vector and scalar potentials are linear functions of the transverse coordinate x only, electric and magnetic strengths are homogeneous.

Thus, orthogonal electric and magnetic fields act on the orbital electron of relativistic H-like ion in the ion rest frame.

3. – Combined Stark-Zeeman effect

To describe the interaction of the orbital electron with the external fields we extend our theoretical model [3] by adding the magnetic field. The wave functions ψ_i and energies ϵ_i of electronic states in the rest frame of the relativistic H-like ion are defined by the Shrödinger equation:

$$(H_0 + U_{\mathrm{SO}} + H_m - |e|\phi')\psi_i = \epsilon_i \psi_i, \tag{1}$$

where H_0 is the Hamiltonian of Bohr's H-like ion, operator U_{SO} describes the spin-orbital coupling (see, for example, in [4]), $-|e|\phi'$ is the potential energy of the orbital electron in the external electric field, H_m is the operator of the orbital electron interaction with the magnetic field:

$$H_m = \frac{|e|}{2mc}(\hat{\mathbf{p}}\mathbf{A}' + \mathbf{A}'\hat{\mathbf{p}}) + \left(\frac{|e|\hbar}{mc}\right)\mathbf{H}'\hat{\mathbf{S}}. \tag{2}$$

In eq. (2) $\hat{\mathbf{p}}$ is the operator of electron momentum, $\hat{\mathbf{S}}$ is the operator of the electron spin written using the Pauli matrices, $|e|$ and m are the absolute value of the charge and the mass of the electron, respectively, and $\hbar$ is the Planck constant.

Equation (1) is solved approximately using the perturbation theory [1, 3]. For the heavy H-like ions, with the high charge number Z, the Lamb shift of s-levels (which is large in this case) has to be taken into account. In our calculation we introduced the operator U_{L} in eq. (1). For the operator U_{L} the diagonal matrix elements corresponding to s-states equal values of the Lamb shifts obtained by using well-known formulas (see, for example, in [5]), and other matrix elements were suggested to be zero.

As the magnetic field is proportional to γv, large energies are required to observe the combined Stark-Zeeman effect. For non-relativistic ions the magnetic field is small and the Zeeman effect is negligible, the position of the electronic energy levels is defined by the Stark effect only, *i.e.* the electronic energy levels are shifted and are split into some sub-levels distinguished by the absolute value of total angular momentum projection onto the electric field direction. For example, the ground state of H-like ion is not split and is shifted only, the first excited state (if the Lamb shift is not taken into account) is split into two sub-levels: the first contains $2s$, $2p_{1/2}$ states and the second $2p_{3/2}$ state. If the ion velocity becomes relativistic, the magnetic field strength grows. Hence, the Zeeman effect should appear. The ordinary Zeeman effect is the splitting of the electronic energy levels in dependence on the projection of total angular momentum onto the direction of magnetic field. For example, the ground state of H-like ion is split into 2 components and the first excited state is split onto 6 components. Obviously, when the magnetic field is added to the electric field, the structure of electronic energy levels caused by the Stark effect should be changed significantly. Also, it should be mentioned that, since electric and magnetic fields are orthogonal, the projection of the total angular momentum onto the direction of any field now is not the quantum number corresponding to any conservation low. Finally, taking into account the Lamb shift results in the gap between energies of $2s_{1/2}$ and $2p_{1/2}$ states.

Let us demonstrate a combined Stark-Zeeman effect for the electronic energy levels of $11\,\mathrm{GeV}/u$ ($\gamma = 12.8$) U^{91+} ions moving in the electric field directed orthogonal to the ion velocity. The Lamb shifts of s-states are significant for these ions: it is more than $10\,\mathrm{eV}$, and must be taken into account. The combined Stark-Zeeman effect appears only in very strong electric field (see, in fig. 2), but this effect changes the structure of electronic energy levels caused by the Stark effect completely. In fig. 2 instead of the influence of Stark effect on the ground and the first excited energy levels described above (4 components, including the Lamb splitting of the $2s_{1/2}$ and $2p_{1/2}$ states) one can see actually the Zeemam structure (10 components). Due to the existence of the electric

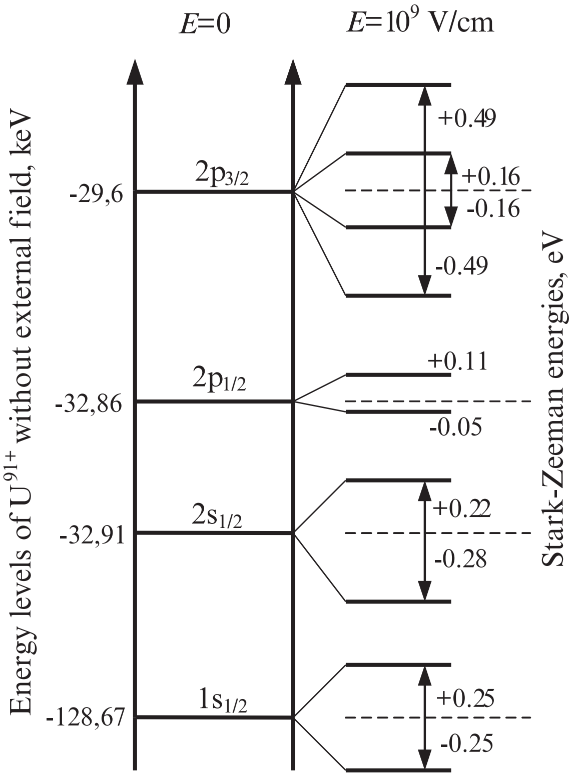

Fig. 2. – The ground and the first excited electronic energy levels in the rest frame of $11\,\mathrm{GeV}/u$ ($\gamma = 12.8$) U^{91+} ions, when the electric field is absent, and when a strong electric field (strength $E = 10^9\,\mathrm{V/cm}$) is present in the laboratory frame. Stark-Zeeman energy is the additional energy, which the level gains due to the combined Stark-Zeeman effect. The splitting of $2s_{1/2}$ and $2p_{1/2}$ levels is caused by the Lamb shift.

field, the shift of sub-levels with respect to the positions of the levels when $E = 0$, is not symmetrical, contrary to the ordinary Zeeman effect.

Thus, the electronic energy level of an ion gains the additional energy due to the combined Stark-Zeeman effect. Let us name this additional energy "the Stark-Zeeman energy". As is shown in fig. 2, the Stark-Zeeman energies are quite small and a strong electric field is required to observe this combined effect. Nevertheless, in [1] it was shown that the Stark-Zeeman energy splitting for ultra-relativistic H-like ions moving in the crystal under planar channeling conditions are comparable to the energies pointed out in fig. 2. Indeed, the planar channeled ion moves with relativistic velocity at a small angle to the crystallographic planes, the continuous planar potential forms an electric field which is orthogonal to the relativistic longitudinal velocity of the ion (the projection of the ion velocity onto the crystallographic planes limiting the planar channel). So, the geometry in the case of the planar channeling of relativistic ions is similar to the matter of this paper. In table I some values of the Stark-Zeeman energies for the ground state and the first excited state are shown. The calculations were carried out for $11\,\mathrm{GeV}/u$ U^{91+} ions planar channeled along the (220) planes of Si crystal. In the case of planar

TABLE I. – *Stark-Zeeman energies for the ground and the first excited electronic energy levels in the case of* (220) *Si channeling of* 11 GeV/u U^{91+} *ions.*

Level	Stark-Zeeman energy (eV)
$2p_{3/2}$	2.25
	1.75
	−0.75
	−2.15
$2p_{1/2}$	1.3
	1.25
$2s_{1/2}$	1.85
	−1.7
$1s_{1/2}$	0.95
	−0.95

channeled ions, the combined Stark-Zeeman effect depends on the distance of the ion from the center of the channel. In general, Stark-Zeeman energies have the maximal absolute values at distances about $0.4a_x$ from the center of the channel (a_x is the distance between neighboring planes limiting the channel), and have the minimal absolute values near the channel center, where the electric field $E \approx 0$. The values, shown in table I, are taken near the points in the channel where the Stark-Zeeman energy for the corresponding sublevel has the maximal absolute value (see [1] for details). Also, it should be underlined that the electric and magnetic fields are not homogeneous in this case. In particular, the inhomogeneity is responsible for the fact that both Stark-Zeeman energies of the $2p_{1/2}$ state in table I are positive, whereas for other energy levels both positive and negative values are presented in table I.

4. – Conclusion

The results of calculations demonstrate that electronic energy levels of ultra-relativistic H-like ions moving in an electric field orthogonal to the ion velocity are shifted and are split due to the combined Stark-Zeeman effect arising in the rest frame of the ion due to the interaction of orbital electron with crossed electric and magnetic fields.

It should be mentioned here that the magnetic field in the rest frame of the ion does not appear, if the external electric field is parallel to the ion velocity. This fact means that the combined Stark-Zeeman effect is absent, when the electric field is used to accelerate the ion. In this case the ordinary Stark effect is observed.

Although very strong electric fields are necessary to observe the combined Stark-Zeeman effect, and the gained additional energies in the homogeneous electric fields are quite small, one can find the conditions where this effect can manifest itself. We mean the effect, which is called resonant coherent excitation (RCE) of ultra-relativistic very heavy H-like ions, *e.g.*, uranium, channeled in a crystal (see, [1] and references therein). The splitting and the shift of the electronic energy levels change the transition frequencies between the energy levels. Therefore, on can conclude that the condition of RCE appearance and other characteristics of RCE (charge state, radiation spectrum and so on) change also.

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